"Hilbert's proof of the finiteness when K is the field of G-invariant functions, $G = SL(m), char(k) = 0$ is so very elegant and simple that it should really be part of every mathematicians bag of tricks."

—David Mumford

The Trivial Notions Seminar

Proudly Announces

Hilbert's Fourteenth Problem

A talk by

Carl Erickson

Abstract

Hilbert's fourteenth problem was originally stated as follows: let k be a field and let K be a subfield of the rational functions $k(x_1, \ldots, x_n)$. Now K is finitely generated as a field, but is $K \cap k[x_1, \ldots, x_n]$ finitely generated as a ring over k?

The motivation for this question came from Hilbert's positive result where k has characteristic 0 and K is the field of rational functions on the vector space k^n invariant by a linear action of the group $SL(m)$. The question of finite generation of rings of invariants of group actions is the main point of interest for this question, since moduli problems of wide concern are often described by the quotient spaces of a variety by an algebraic group. We would like our quotient spaces to be varieties! We will begin by discussing a few examples of these moduli problems and group actions.

Then we will mention Nagata's negative solution to this problem, found in 1959. However, the rest of the talk will introduce and focus on the notions of reductivity of an algebraic group G over k - reductivity, linear reductivity, and geometric reductivity - their relation to each other, and their role in representation theory and geometry. This will amount to a fractional introduction to geometric invariant theory, developed by Mumford and his collaborators in the 1960s to construct quotient spaces and address moduli problems.

Thursday November 18^{th}, at 3:00 pm
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