3/7/05 QUIZZ NR 5	Math118, O.Knill

Name:

1) How many equilibrium points does the Lorenz system have in total for $r > 1$, when $\sigma > 0, b > 0$ are fixed?

a) three.
b) two.
c) one.
d) none.

2) At the parameter $r = r_1 = 24.74 = 470/19$, at each of the two additional equilibrium points $C^{\pm}$ of the Lorenz system, something happens, when the parameter r increases:

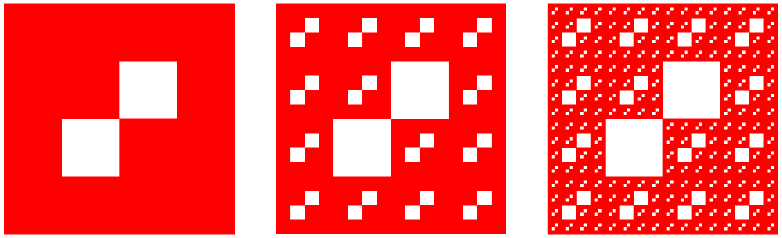
a) A sub-critical Hopf bifurcation: an unstable limit cycle collides with the critical point.

b) A Hopf bifurcation: a stable equilibrium point becomes unstable and ejects a limit cycle.

c) A flip bifurcation: the equilibrium point $C^{\pm}$ double and undergo a pitchfork bifurcation.

d) A period doubling bifurcation for cycles: periodic cycles double.

3) We define an object in the plane similar to the Shirpinski carpet by cutting away 2 squares of length 1/4 from a square of length 1 and repeating this construction with remaining squares of length 1/4 etc: the first three steps are shown below:



What is the dimension of this object?

a) $\log(14)/\log(4)$
b) $\log(5)/\log(3)$
c) $\log(20)/\log(5)$

4) Find the box counting dimensions of the following sets:

a) The graph of the function $f(x) = \sin(x)$ in the plane.
b) A filled triangle.
c) The set $\{1, 1/2, 1/3, 1/4, 1/5, 1/6, ...\}$
d) The Cantor set.

5) Which of the following properties does a strange attractor K of a differential equation in space possess:

a) sensitive dependence on initial conditions.
b) the dimension must be a non-integer.
c) The set K has to be an attractor.
d) The set has to contain a sink, (this is an equilibrium point for which all eigenvalues have negative real part).

6) Which of the following differential equations in space produces a volume preserving flow?

a)

$$\begin{aligned} \dot{x} &= a(y-x) \\ \dot{y} &= cx - xz - y \\ \dot{z} &= xy - bz \end{aligned}$$

b)

$$\begin{aligned} \dot{x} &= -(y+z) \\ \dot{y} &= x + 0.2y \\ \dot{z} &= 0.2 + xz - cz \end{aligned}$$

c)

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -y - x + x^3 - c\cos(z) \\ \dot{z} &= 1 \end{aligned}$$

d)

$$\begin{aligned} \dot{x} &= a\sin(z) + c\cos(y) \\ \dot{y} &= b\sin(x) + a\cos(z) \\ \dot{z} &= c\sin(y) + b\cos(x) \end{aligned}$$