

Name:

1) Which of the following properties do hold at a **flip bifurcation**?

- a) $f'(x) = 1$.
- b) $f'(x) = -1$.
- c) $(f^2)'(x) = 1$.
- d) $(f^2)'(x) = -1$.
- e) The bifurcation is also called **pitch-fork** bifurcation.

2) Which of the following properties do hold at a **saddle node bifurcation**?

- a) $f'(x) = 1$.
- b) $f'(x) = -1$.
- c) $(f^2)'(x) = 1$.
- d) $(f^2)'(x) = -1$.
- e) The bifurcation is also called **blue-sky** bifurcation.

3) The tent map $g(x) = 1 - 2|x - 1/2|$ is conjugated to the logistic map $f_c(x) = cx(1 - x)$ for the parameter

- a) $c = 0$.
- b) $c = 1$.
- c) $c = 2$.
- d) $c = 3$.
- e) $c = 4$.

4) Which of the following formulas do give the Lyapunov exponent of an orbit x_0, x_1, x_2, \dots of a map $f : [0, 1] \rightarrow [0, 1]$?

- a) $\lambda(f, x_0) = \lim_{n \rightarrow \infty} \log |(f^n)'(x_0)|$
- b) $\lambda(f, x_0) = \lim_{n \rightarrow \infty} \frac{1}{n} \log |(f^n)'(x_0)|$
- c) $\lambda(f, x_0) = \lim_{n \rightarrow \infty} \frac{1}{n} |(f^n)'(x_0)|$
- d) $\lambda(f, x_0) = \lim_{n \rightarrow \infty} \frac{1}{n} (f^n)'(x_0)$

5) Which properties can change at a bifurcation point?

- a) the number of periodic points.
- b) the stability of periodic points.
- c) the sign of $\log |f'(x)|$.
- d) the sign of $f'(x)$.

6) The number of periodic points (of any period) of the logistic map $f_c(x) = cx(1 - x)$ is always finite.

- a) True
- b) False

The number of periodic points of a fixed period n of the logistic map $f_c(x) = cx(1 - x)$ is always finite.

- a) True
- b) False

7) The topological entropy is a measure for the exponential growth of the number of periodic points. There are parameter values for the map $f_c(x) = cx(1 - x)$ for c between 0 and 1 for which the topological entropy is positive.

- a) True
- b) False

For $c > 4$, the logistic map $f_c(x) = cx(1 - x)$ is a map on the interval $[0, 1]$.

- a) True.
- b) False.

8) Which of the following things are true:

- a) $1/2$ is an eventually periodic orbit of the Ulam map $f(x) = 4x(1 - x)$.
- b) $1/2$ is a periodic orbit of the Ulam map.
- c) The Lyapunov exponent $\lambda(f, 1/2)$ is equal to $\log(2)$ if f is the Ulam map.