2/2/05 QUIZZ NR 0 **Math118, O.Knill**

ABSTRACT. This page should give you an idea, how weekly multiple choice quizzes look like. The quizzes do not any special preparation if you follow the lectures. Several choices in the multiple choice part are possible. The questions below address two lectures. The lecture today as well as the lecture on Friday. Next Monday, we already have a quizz of this format.

1) How many midterms do we have in Math 118r?

a) None, we have quizzes.
b) One midterm
c) Two midterms

2) Check whatever belongs to the theory of dynamical systems:

a) A group acting on a set.
b) Predict the future of systems and explore the limitations of this predictions.
c) Compute square roots of real numbers.
d) Understand the iteration of maps.

3) Look at the map $T(x) = x^2 + x$. Which of the following sequences form an **orbit** of x through $x = 1$:

a) $1, 6, 12, 20, 30, ...$
$b)$1,2,5,30,...
c) $0, 0, 0, 0, 0, ...$
$d)$2,5,30,930,...

4) Which of the following dynamical systems have a discrete time? We replace "map" or "differential equation" with "system".

a) Henon system
b) Van der Pool system
c) Standard system
d) Geodesic system
e) Billiard system.
f) Digits of π system.
g) Cellular automata system

5) Which of the following dynamical systems is the **Lorentz system**

a) $\ddot{x} + x + (x^2 - 1)y = 0$.
b)

$$\begin{aligned} \dot{x} &= 10(y - x) \\ \dot{y} &= -xz + 28x - y \\ \dot{z} &= xy - \frac{8z}{3} \end{aligned}$$

c) $T \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10(y - x) \\ z + 28x - y \\ xy - \frac{8z}{3} \end{bmatrix}$.

6) What is a semigroup?

a) A set G with an operation $*$.
b) A set G with an operation $*$ such that $(x \star y) \star z = x \star (y \star z)$.
c) A set G with an operation $*$ such that $(x \star y) \star z = x \star (y \star z)$ with a neutral element e satisfying $x \star e = x$.
d) A set G with an operation $*$ such that $(x \star y) \star z = x \star (y \star z)$ with a neutral element e satisfying $x \star e = x$ and such that for every x, there is a y such that $x \star y = e$.

7) Which of the following sets are semigroups?

a) The natural numbers.
b) The set of words over a finite alphabet with the operation $v \star w = vw$ of putting the words together.
c) The set of all subsets of a finite set with the operation $A \star B = A \cup B$.

8) Which of the following dynamical systems allow a numerical computation of the square root of 7:

a) $T(x, y) = ((x + y)/2, 2xy/(x + y))$.
b) $T(x) = \sqrt{x} - 7$.
c) $T(x) = x^2 + 7$.

9) How could dynamical systems theory helped to save lives. Check each which apply:

a) Predict wave heights from the strength of earthquakes triggering tsunamis.
b) Predict the outcome of the lotto.
c) Predict the sector in which the roulette ball falls.
d) Predict the global warming on earth.

2/7/05 QUIZZ NR 1	**Math118, O.Knill**

Name:
Email:

1) Look at the map $T(x) = x^2 + 1$ on the real line. Which of the following sequences form an **orbit** of x with initial condition $x_0 = 0$:

a) $0, 1, 2, 3, 4, ...$

b) $0, 1, 2, 5, 26, ...$

c) $0, 0, 0, 0, 0, ...$

d) $1, 1, 1, 1, ...$

2) Which of the following orbits are periodic cycles of the dynamical system?

a) $(x(t), y(t)) = (\sin(t), \cos(t))$ for the harmonic oscillator differential equation $\frac{d}{dt}x(t) = y(t), \frac{d}{dt}y(t) = -x(t)$.

b) The longest diagonal in a convex billiard table.

c) A single alive cell in the game of life.

d) The point 0 in the logistic map $T(x) = 4x(1 - x)$.

3) Which of the following dynamical systems have a discrete time? We replace "map" or "differential equation" with "system".

a) The game of life

b) The Lorentz system

c) The billiard system.

d) The harmonic oscillator system $\frac{d}{dt}x = y, \frac{d}{dt}x = -x$.

4) There is a sentence attributed to Steven Smale which appears also in the movie "Jurassic Park". The statement is "The wing of a butterfly in X can produce a tornado in Y a few weeks later":

a) X=Rio, Y=Texas
b) X=New York, Y = Los Angeles
c) X=Chicago, Y = New Orleans

5) We have seen the map

$$T(x, y) = ((x + y)/2, 2xy/(x + y))$$

to compute the square root of numbers. The number $2xy/(x + y)$ is called the

a) The geometric mean.
b) The algebraic mean.
c) The harmonic mean.

of x and y.

6) We have seen taht for a computer, iterations of the map $T(x) = 4x(1 - x)$ or the map $S(x) = 4x - 4x^2$ give different results for $T^n(x)$ and $S^n(x)$ if n is large and this happened for identical initial condition x_0. For which n, did we see different results?

a) $n = 1$
b) $n = 10$
c) $n = 100$

2/14/05 QUIZZ NR 2	**Math118, O.Knill**

Name:

1) Which of the following properties do hold at a **flip bifurcation**?

a) $f'(x) = 1$.
b) $f'(x) = -1$.
c) $(f^2)'(x) = 1$.
d) $(f^2)'(x) = -1$.
e) The bifurcation is also called **pitch-fork** bifurcation.

2) Which of the following properties do hold at a **saddle node bifurcation**?

a) $f'(x) = 1$.
b) $f'(x) = -1$.
c) $(f^2)'(x) = 1$.
d) $(f^2)'(x) = -1$.
e) The bifurcation is also called **blue-sky** bifurcation.

3) The tent map $g(x) = 1 - 2|x - 1/2|$ is conjugated to the logistic map $f_c(x) = cx(1-x)$ for the parameter

a) $c = 0$.
b) $c = 1$.
c) $c = 2$.
d) $c = 3$.
e) $c = 4$.

4) Which of the following formulas do give the Lyapunov exponent of an orbit $x_0, x_1, x_2, \ldots$ of a map $f : [0,1] \to [0,1]$?

a) $\lambda(f, x_0) = \lim_{n\to\infty} \log|(f^n)'(x_0)|$
b) $\lambda(f, x_0) = \lim_{n\to\infty} \frac{1}{n} \log|(f^n)'(x_0)|$
c) $\lambda(f, x_0) = \lim_{n\to\infty} \frac{1}{n} |(f^n)'(x_0)|$
d) $\lambda(f, x_0) = \lim_{n\to\infty} \frac{1}{n} (f^n)'(x_0)$

5) Which properties can change at a bifurcation point?

a) the number of periodic points.
b) the stability of periodic points.
c) the sign of $\log|f'(x)|$.
d) the sign of $f'(x)$.

6) The number of periodic points (of any period) of the logistic map $f_c(x) = cx(1-x)$ is always finite.

a) True
b) False

The number of periodic points of a fixed period n of the logistc map $f_x(x) = cx(1-x)$ is always finite.

a) True
b) False

7) The topological entropy is a measure for the exponential growth of the number of periodic points There are parameter values for the map $f_c(x) = cx(1-x)$ for c between 0 and 1 for which the topological entropy is positive.

a) True
b) False

For $c > 4$, the logistic map $f_c(x) = cx(1-x)$ is a map on the interval $[0,1]$.

a) True.
b) False.

8) Which of the following things are true:

a) 1/2 is an eventually periodic orbit of the Ulam map $f(x) = 4x(1-x)$.
b) 1/2 is a periodic orbit of the Ulam map.
c) The Lyapunov exponent $\lambda(f, 1/2)$ is equal to $\log(2)$ if f is the Ulam map.

2/23/05 QUIZZ NR 3 **Math118, O.Knill**

Name:

1) One of the following maps is an area preserving Henon map. Which one?

a) $T(x, y) = (3(1 - x^2) - y, 2x)$
b) $T(x, y) = (2x - y + \sin(x), x)$.
c) $T(x, y) = (2x - y + \sin(x), x/2)$.
d) $T(x, y) = (3(1 - x^2) - y, x)$

2) For a certain parameters c, the Standard map $T(x, y) = (2x + c\sin(x) - y, x) \mod 1$ is integrable. Which integral verifies this fact?

a) $F(x, y) = x - y$.
b) $F(x, y) = x$
c) $F(x, y) = x^2 + y^2$.
d) $F(x, y) = 1$.

3) Which of the following matrices is the Jacobean matrix of the transformation $T(x, y) = \begin{bmatrix} (x^2 + y^2)/2 \\ 2x^2 - y^2 \end{bmatrix}$?

a) $DT(x, y) = \begin{bmatrix} x & y \\ 4x & -2y \end{bmatrix}$.

b) $DT(x, y) = \begin{bmatrix} x^2/2 & y^2/2 \\ 2x^2 & -y^2 \end{bmatrix}$.

4) The map T of the previous problem is area preserving.

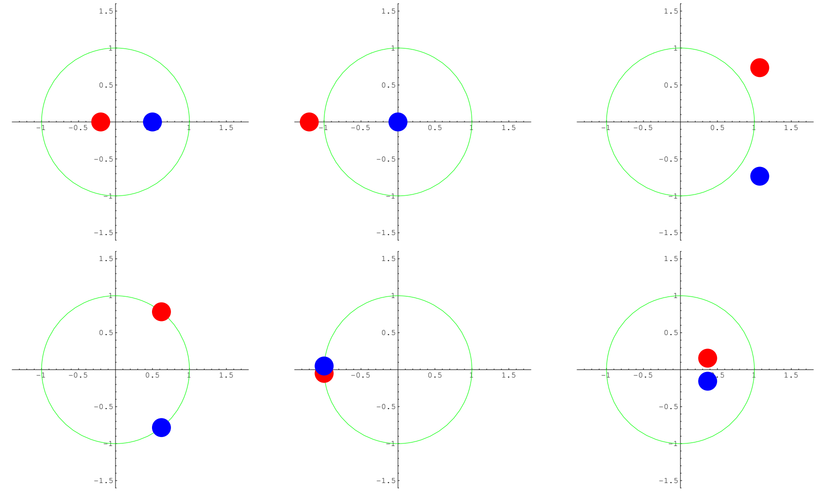
a) True

b) False

5) A map in the plane is called an **involution** if $T^2 = Id$, that is if every point is periodic with period 2. Which of the following statements are true?

a) In general, an involution is integrable.
b) The map $T(x, y) = (-x, y + c\sin(x))$ is an involution.
c) All linear involutions are area-preserving.
d) The map $T(x, y) = \begin{cases} (-x/2, y) & x > 0 \\ (-2x, y) & x < 0 \end{cases}$ is an involution.
e) In general, an involution is area preserving.

6) Which of the following 6 pictures shows the eigenvalues of a Jacobean at a fixed point of a map T in the plane which has stable and unstable manifolds:



7) If the stable and unstable manifolds of a hyperbolic fixed point (x_0, y_0) of intersect transversely, then this intersection point is called

a) an equilibrium point
b) an integral
c) a homoclinic point
d) a periodic point
e) a horse shoe

8) Which of the following facts are true about the Henon attractor, obtained with parameters $a = 1.4, b = 0.3$?

a) It contains the stable manifold of one of the hyperbolic fixed points.
b) It contains a horse shoe.
c) It contains infinitely many periodic points.
d) It is integrable.

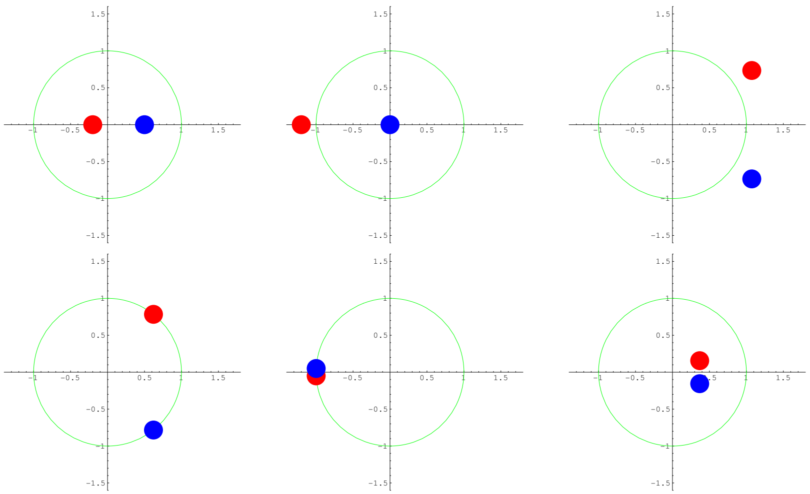
2/28/05 QUIZZ NR 4	Math118, O.Knill

Name:

1) Which of the following differential equations produces an area-preserving flow?

a) $\frac{d}{dt}x = x + y^2, \frac{d}{dt}y = -y + x^2$
b) $\frac{d}{dt}x = -x + x^2, \frac{d}{dt}y = y - y^3$
c) $\frac{d}{dt}x = 1, \frac{d}{dt}y = 2$
d) $\frac{d}{dt}x = y^2, \frac{d}{dt}y = x^2$

2) Which of the following 6 pictures of eigenvalues a Jacobean $DF(x_0, y_0)$ at an equilibrium point (x_0, y_0) which is stable?



3) What happens at a Hopf bifurcation?

a) A pair of eigenvalues crosses the unit circle.
b) A pair of eigenvalues crosses the imaginary axes.
c) A single eigenvalues crosses the unit circle.
d) A single eigenvalues crosses the imaginary axes.
e) An attractive equilibrium point becomes repelling.

4) Which of the following formulations is the Poincare Bendixson theorem?

a) An orbit in the plane which stays in a bounded region is either asymptotic to an equilibrium point or to a limit cycle.
b) An orbit in the plane which is not asymptotic to a limit cycle is attracted to an equilibrium point.
c) Every orbit of a differential equation in the plane is either asymptotic to a limit cycle or to an equilibrium point.

5) A differential equation of the form $\frac{d}{dt}x(t) = H_y(x, y), \frac{d}{dt}y(t) = -H_x(x, y)$, where $H(x, y)$ is a function of two variables.

a) produces an area-preserving flow.
b) is integrable.
c) has an attractive limit cycle.
d) has at least one attractive equilibrium point.

6) Which of the following differential equations is called the **van der Pol oscillator** ?

a) $\frac{d^2}{dt^2}x = -x$.
b) $\frac{d^2}{dt^2}x + c(x^2 - 1)\frac{d}{dt}x + x = 0$.
c) $\frac{d^2}{dt^2}x + c(x^2 - 1)\frac{d}{dt}x + \sin(x) = 0$.

7) Lienard systems have

a) exactly one repelling limit cycle.
b) exactly one attracting limit cycle.
c) exactly one repelling equilibrium point.
d) exactly one attractig equilibrium point.

3/7/05 QUIZZ NR 5 **Math118, O.Knill**

Name:

1) How many equilibrium points does the Lorenz system have in total for $r > 1$, when $\sigma > 0, b > 0$ are fixed?

a) three.
b) two.
c) one.
d) none.

2) At the parameter $r = r_1 = 24.74 = 470/19$, at each of the two additional equilibrium points $C^{\pm}$ of the Lorenz system, something happens, when the parameter r increases:

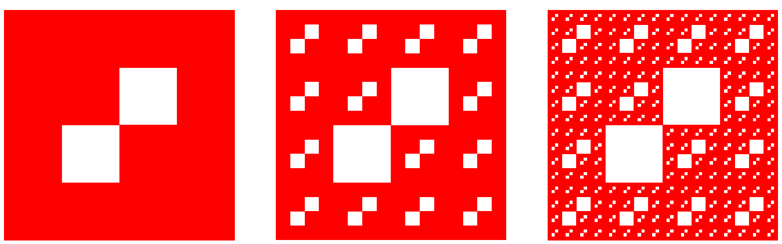
a) A sub-critical Hopf bifurcation: an unstable limit cycle collides with the critical point.

b) A Hopf bifurcation: a stable equilibrium point becomes unstable and ejects a limit cycle.

c) A flip bifurcation: the equilibrium point $C^{\pm}$ double and undergo a pitchfork bifurcation.

d) A period doubling bifurcation for cycles: periodic cycles double.

3) We define an object in the plane similar to the Shirpinski carpet by cutting away 2 squares of length 1/4 from a square of length 1 and repeating this construction with remaining squares of length 1/4 etc: the first three steps are shown below:



What is the dimension of this object?

a) $\log(14)/\log(4)$
b) $\log(5)/\log(3)$
c) $\log(20)/\log(5)$

4) Find the box counting dimensions of the following sets:

a) The graph of the function $f(x) = \sin(x)$ in the plane.
b) A filled triangle.
c) The set $\{1, 1/2, 1/3, 1/4, 1/5, 1/6, ...\}$
d) The Cantor set.

5) Which of the following properties does a strange attractor K of a differential equation in space possess:

a) sensitive dependence on initial conditions.
b) the dimension must be a non-integer.
c) The set K has to be an attractor.
d) The set has to contain a sink, (this is an equilibrium point for which all eigenvalues have negative real part).

6) Which of the following differential equations in space produces a volume preserving flow?

a)

$$\begin{aligned} \dot{x} &= a(y - x) \\ \dot{y} &= cx - xz - y \\ \dot{z} &= xy - bz \end{aligned}$$

b)

$$\begin{aligned} \dot{x} &= -(y + z) \\ \dot{y} &= x + 0.2y \\ \dot{z} &= 0.2 + xz - cz \end{aligned}$$

c)

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -y - x + x^3 - c\cos(z) \\ \dot{z} &= 1 \end{aligned}$$

d)

$$\begin{aligned} \dot{x} &= a\sin(z) + c\cos(y) \\ \dot{y} &= b\sin(x) + a\cos(z) \\ \dot{z} &= c\sin(y) + b\cos(x) \end{aligned}$$

3/14/15926... QUIZ NR 6 **Math118, O.Knill**

Name:

1) The billiard in an ellipse is known to be integrable. What is the integral F?
a) The sum of the shortest distances of the trajectory line to the focal points.
b) The product of the shortest distances of the trajectory line to the focal points.
c) The impact angle θ.
d) The distance of the focal points.

2) A billiard table γ is obtained by doing the string construction to a convex set K. (For example, in the homework for today, K was a triangle or square):
Check all which applies:

a) The billiard has a caustic.

b) The billiard map can not have glancing trajectories: trajectories for which the angle θ can become arbitrarily close to 0 and arbitrarily close to π. .

c) The billiard map has periodic orbits of period 17.

d) The billiard map has an invariant curve in the annulus $R/Z \times [-1,1]$.

3) For which coordinates is the billiard map area-preserving?

a) The (s, θ) coordinates, where s is the arc length normalized that the table has length 1 and where θ is the impact angle.

b) The (x, y) coordinates, where x is the arc length normalized so that the table has length 1 and where $y = \cos(\theta)$.

c) The (s, s') coordinates, where (s, s') are successive impact points of the trajectory and where s is the arc length parameter.

4) Every strictly convex smooth Birkhoff billiard has periodic orbits, because

a) We can maximize the length functional of the polygon.

b) We can minimize the length functional of the polgyon.

c) We can maximize the area functional inside a polygon.

d) We can minimize the area functional inside a polygon.

5) Which of the following are open mathematical problems?

a) Every billiard in a triangle has a periodic orbit.

b) Every exterior billiard has the property that for (x, y) outside the table, $T^n(x, y) \to \infty$.

c) The solar system is stable in the sense that all planets remain in a bounded region near the sun for all times.

d) There exists a convex billiard for which the Lyapunov exponent is positive on a set of positive area.

e) There exists a smooth convex billiard for which there are no glancing orbits.

6) Which of the following equations is called the **Euler equation**? We use the notation $h_1(x, y) = \frac{\partial}{\partial x} h(x, y)$ and $h_2(x, y) = \frac{\partial}{\partial y} h(x, y)$. Just one answer is correct.

a) $h_1(x_{i-1}, x_i) + h_2(x_i, x_{i+1}) = 0$.

b) $h_2(x_{i-1}, x_i) + h_1(x_i, x_{i+1}) = 0$.

c) $h_1(x_{i-1}, x_i) + h_1(x_i, x_{i+1}) = 0$.

d) $h_2(x_{i-1}, x_i) + h_2(x_i, x_{i+1}) = 0$.

e) $h_1(x_{i-1}, x_i) - h_2(x_i, x_{i+1}) = 0$.

f) $h_2(x_{i-1}, x_i) - h_1(x_i, x_{i+1}) = 0$.

g) $h_1(x_{i-1}, x_i) - h_1(x_i, x_{i+1}) = 0$.

h) $h_2(x_{i-1}, x_i) - h_2(x_i, x_{i+1}) = 0$.

7) Which of the following matrices is conjugated to the Jacobean $DT(x_i, y_i)$ of the billiard map. l_i is the length of the trajectory before the impact with the boundary where the impact angle is θ_i and the curvature is κ_i.

a) $B_i = \begin{bmatrix} 1 & 0 \\ -\frac{2\kappa_i}{\sin(\theta_i)} & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & l_i \\ 0 & 1 \end{bmatrix}$

b) $B_i = \begin{bmatrix} 1 & l_i \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ -\frac{2\kappa_i}{\sin(\theta_i)} & 1 \end{bmatrix}$

c) $B_i = \begin{bmatrix} 1 & 0 \\ \frac{2\kappa_i}{\sin(\theta_i)} & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -l_i \\ 0 & 1 \end{bmatrix}$.

8) Today is Pi-day. Somebody cuts a piece from a circular apple pie. We use the remaining part as a billiard table. Which of the following is true:

a) The table is convex.

b) The table is not convex.

3/21 (Start of Spring!) QUIZ NR 7 **Math118, O.Knill**

Name:

1) Which of the following formulations is the full content of the Hedlund-Lyndon-Curtis theorem? (only one answer is correct).

a) Any map T on $X = \{0,1\}^Z$ which is continuous and commutes with the shift is of the form $T(x)_n = \phi(x_{n-1}, x_n, x_{n+1})$.

b) A cellular automaton is a continuous map on $X = \{0,1\}^Z$.

c) A shift commuting, continuous map on $X = \{0,1\}^Z$ is a cellular automaton.

d) Any continuous map on $X = \{0,1\}$ has the property that the n'th entry of $T(x)$ depends only on finitely many neigbors.

2) True or False?

a) There exists a cellular automaton T such that the set of periodic orbits is dense.

a) There exists a cellular automaton T such that the set of periodic orbits of period 11 are dense.

b) There exists a cellular automaton T such that $\{T^n(x), n = 1, 2, ...\}$ covers the entire set $X = \{0,1\}^Z$.

c) There exists a cellular automaton T, such that $\{T^n(x), n = 1, 2, ..., \}$ is dense in X.

3) We have $T(x)_n = x_{n+1} + x_{n-1} + x_n$ mod(2). What is the image of the sequence $x = (..., 1, 0, 1, 0, 1, 0, 1, ...)$?

a) (...,1,1,1,1,1,1,1,1,1,1,1,...)

b) (...,0,0,0,0,0,0,0,0,0,0,0,...)

c) (...,1,0,1,0,1,0,1,0,1,0,1....)

4) Assume a cellular automaton has the property that $T^3(x)$ is the shift. Which of the following statements are true?

a) T is chaotic in the sense of Devaney.

b) T^3 is chaotic in the sense of Devaney.

c) T^9 is chaotic in the sense of Devenay

5) If $x = (...1, 0, 1, 0, 1, 0, 1, 0, 1, 0, ...$ with $x_0 = 1$ and $y = (..., 0, 0, 0, 0, 0, ...)$, then the distance between these two points $d(x, y)$ is

a) 1
b) 1/2
c) 0
d) 2

6) True or false?

If $d(x, y) = 1/10$, then $d(\sigma(x), \sigma(y)) = 1/10$, where σ is the shift.

7) A lattice gas cellular automaton

a) conserves the total momentum of the particles

b) is used to simulate fluids

c) is used to simulate sand dynamics.

d) conserves the total angular momentum of the particles.

e) conserves the total energy of the particles.

8) What is a "glider" in the game of life (X, T)?

a) A configuration x which satisfies $T^n(x) = \sigma^m(x)$ for $n, m > 0$.

b) A configuration with finitely many living cells which satisfies $T^n(x) = \sigma^m(x)$ for $n, m > 0$.

c) A configuration which satisfies $T^n(x) = x$ for $n > 0$.

d) A fixed point of T.

9) Who is believed to have first come up with the notion of cellular automata?

a) Hedlund at Harvard

b) Wolfram at Caltech

b) Ulam and von Neuman at Los Alamos

10) If you allow the alphabet of a cellular automaton to become a continuum, then the corresponding dynamical system is called a

a) partial differential equation.

b) coupled map lattice.

c) map on an infinite dimensional space

d) an infinite system of coupled ordinary differential equations.

11) (5 points if correct) A one dimensional automaton maps x to y, where

x =	...	1	1	1	0	1	1	0	0	1	1	0	0	0	0	1	0	...
y =	...	1	0	0	1	1	0	0	0	1	0	0	1	1	0	0	1	...

What is the Wolfram number of this cellular automaton?

4/4 QUIZ NR 8	**Math118, O.Knill**

Name:

1) With $f_c(z) = z^2 + c$, which of the following statements are true?

a) The Mandelbrot set is the set of complex numbers such that $f_c^n(c) \to \infty$.

b) The Mandelbrot set is the set of complex numbers such that $f_c^n(c) \leq 2$ for all n. (You have shown in the homework that if $f_c^n(c) > 2$ then c is not in the Mandelbrot set).

c) The Mandelbrot set is the set of complex numbers such that $f_c^n(c)$ stays bounded.

2) True or False?

a) Every quadratic polynomial is conjugated to the polynomial $f_c(z) = z^2 + c$.

b) Every cubic polynomial is conjugated to a polynomial $g_c(z) = z^3 + c$.

c) Every cubic polynomial is conjugated to a polynomial $g_{a,b}(z) = z^3 - 3a^2 z + b$.

3) True or False?

a) The union of the Julia set and the Fatou set is the entire complex plane.

b) The Fatou set is the complement of the Mandelbrot set.

4) Who was historically first to have made pictures of the Mandelbrot set?

a) John Hubbard.

b) Douady and Hubbard.

c) Benoit Mandelbrot.

d) Brooks and Matelski.

5) A fixed point of a quadratic map $f(z)$ is defined to be stable, if (only one answer applies):

a) $f'(z) < 1$.

b) $|f'(z)| \leq 1$

c) $|f'(z)| = 0$.

d) $|f'(z)| < 1$.

6) True or False?

a) The Ulam map $f(z) = 4z(1-z)$ is in the complex plane conjugated to $f_{-2}(z) = z^2 - 2$.

b) The Julia set of the polynomial $f_0(z) = z^2$ is the circle with radius 1.

c) The filled in Julia set of the polynomial $f(z) = 4z^2$ is the disc of radius 1/2.

7) True or False? The Ulam map $f(z) = 4z(1-z)$ restricted to its Julia set is conjugated to $x \mapsto 2x \bmod 1$.

8) Which of the following dynamical systems is called the **Newton iteration** to find the root $f(z) = 0$:

a) $T(z) = 1 - f(z)/f'(z)$

b) $T(z) = z - f'(z)/f(z)$

c) $T(z) = 1 - f'(z)/f(z)$

d) $T(z) = z - f(z)/f'(z)$

9) In order to find a fixed point of a map S, we can try to apply the Newton method to one of the following:

a) $T(z) = S(z) - z$

b) $T(z) = S'(z) - z$

c) $T(z) = z - S(z)/S'(z)$.

10) True or False? The Mandelbrot set is a fractal because its dimension has shown to be smaller then 2 and bigger than 1.

4/11 QUIZ NR 9	Math118, O.Knill

Name:

1) Which of the following properties apply to the Baker transformation T on the square $[0,1) \times [0,1)$.

a) The map is continuous.
b) There is a conjugation of the map to a subshift $S(Y) \subset \{0,1\}^Z$
c) There is a conjugation of the map to the shift $S(Y) \subset \{0,1\}^N$
d) The map is area-preserving.
e) The map has many periodic points.
f) The map has no periodic points.
g) The map is invertible.

2) True or False: If you take a subshift X of finite type, and a cellular automaton ϕ, then $\phi(X)$ is a subshift of finite type.

3) True or False: If you take a sophic subshift and a cellular automaton ϕ, then $\phi(X)$ is a sophic subshift.

4) Which of the following inclusions are true? (I had this once wrong on the blackboard and Orr had corrected it):

a) subshifts $\supset$ subshifts of finite type $\supset$ sophic subshifts.

b) subshifts $\supset$ sophic subshifts $\supset$ subshifts of finite type.

5) True or False: the **language** of a subshift of finite type is the set of forbidden words.

6) What can you say about the subshift X of finite type over the alphabet $\{a,b,c\}$ defined by the forbidden words $\{aa, bb, cc, ac, ba, cb\}$?

a) X does not contain any point.
b) X contains only finitely many points.
c) X contains infinitely many points.

7) Which of the following subshifts is the shift over the alphabet $\{a,b\}$ for which all words $bab, baaab, baaaaab, baaaaaaab, baaaaaaaaab, ...$ etc. are forbidden?

a) The Fibonacci shift
b) The even shift
c) The golden mean shift
d) The full shift.

8) When doing symbolic dynamics for the Arnold cat map $T(x,y) = (2x+y, x+y)$ mod 1, one uses a subshift of finite type over an alphabet with a minimal amount of letters. This alphabet has

a) 2 elements.
b) 3 elements.
c) 5 elements.
d) 6 elements.

9) Two random variables Y and Z taking finitely many values are called **uncorrelated** if and only if

a) $P[Y = a, Z = b] = P[Y = a]P[Z = b]$ for all possible numbers a, b.

b) $E[YZ] = E[Y]E[Z]$.

10) Assume, a sequence of independent identically distributed random variables $Y_1, Y_2, Y_3, ...$ describes drawing a card from an infinite deck containing 52 types of cards. It is assumed that each card appears with the same probability 1/52 and that a card can appear multiple times. How do you model these random variables?

a) $Y_k(y) = y_k$, where $y \in \{1, ..., 52\}^N$.

b) $Y_k(y) = k$, where $y \in \{1, ..., 52\}^N$.

c) $Y_k(y) = y$, where $y \in \{1, ..., 52\}^N$.

4/18 QUIZ NR 10 — Math118, O.Knill

Name:

1) What is special about the **golden ratio** $\theta = (\sqrt{5}-1)/2$? Check everything which applies:

a) It has the smallest possible continued fraction expansion $\theta = [a_0; a_1, a_2, a_3, ...]$.
b) The partial fractions p_n/q_n have the property that q_n and p_n grow like Fibonnacci numbers.
c) θ is a solution of $x = 2/(2+x)$.

2) Which of the the following numbers has the continued fraction expansion

$$x = [0; 2, 1, 2, 1, 2, 1, ...] = \frac{1}{2 + \frac{1}{1+\frac{1}{2++....}}}$$

a) x is a solution of $x = 1/(2 + 1/(1 + x))$.
b) x is a solution of $x = 1/(2 + 1/x))$.
c) x is a solution of $x = 1/(1 + 2/x))$.
d) x is a solution of $x = 1/(1 + 1/2 + x))$.

3) Which of the following statements is called Dirichlets theorem for an irrational number α.

a) For every n, there exists $q \leq n$ such that

$$|\alpha - p/q| \leq 1/q^2 .$$

b) For every n, there exists $q \leq n$ such that

$$|\alpha - p/q| \leq 1/q .$$

c) For every q, there exists p such that

$$|\alpha - p/q| \leq 1/q^2 .$$

d) For every q, there exists p such that

$$|\alpha - p/q| \leq 1/q .$$

4) Which of the following three formulas give the correct recursion for the partial fractions p_n/q_n:

a) $p_{n+1} = a_n p_n + p_{n-1}$, $q_{n+1} = a_n q_n + q_{n-1}$.
b) $p_{n+1} = p_n + a_n p_{n-1}$, $q_{n+1} = q_n + a_n q_{n-1}$.
c) $p_{n+1} = a_n p_n + a_{n-1} q_{n-1}$, $q_{n+1} = a_n p_n + a_{n-1} q_{n-1}$

5) The dynamical logarithm problem is the problem

a) to find the time to reach from a point x to a neighborhood of a point y.
b) to find the point y which is reached after time t when starting from x.
c) to find the initial point x, when reaching the point y after time t.

6) Which dynamical system is involved when making a decimal expansion of a real number.

a) $T(x) = 10x$.
b) $T(x) = 10x \bmod 1$.
c) $T(x) = 10/x \bmod 1$.

7) The quadratic map $T(x) = x^2 + c$ is also useful in number theory. Where?

a) to compute the eclipse times in calendars.
b) to factor large integers.
c) to understand why our tonal system has 12 scales between an octave and 19 scales for a perfect fifth .

8) Who came up with the idea to factor integers n by finding two numbers x and y satisfying $x^2 = y^2 \bmod n$ and then having a common nontrivial factor of $x - y$ with n?

a) Fermat.
b) Tchebychev.
c) Minkovsky.

9) When finding lattice points close to graphs of quadratic polynomials, we were led to a dynamical system on the two dimensional torus. This system is

a) $(x, y) \to (x + 2a, x + y) \bmod 1$.
b) $(x, y) \to (2x + y, x + y) \bmod 1$.
c) $(x, y) \to (x + y, x - y) \bmod 1$.
d) $(x, y) \to (ax^2 + bx + c, x) \bmod 1$.

10) Check for whatever continued fractions are useful:

a) To compute eclipse cycles.
b) To justify why we use a 12 scale system in music.
c) To find lattice points close to lines in the plane.
d) To factor integers.

4/25 QUIZ NR 11 **Math118, O.Knill**

Name:

1) True or False: there are always equilibrium solutions to the Newtonian n-body problem.

2) What is the minimal number of bodies for which one can prove that an escape to infinity in finite time is possible?

a) 2
b) 3
c) 4
d) 5

3) Which of the following problems are considered "restricted three body problems":

a) The Earth-Moon-Sun system.
b) The Sitnikov problem.
c) A planet moving in the influence of a uniformly rotating binary star system.
d) The Kepler problem.

4) Who was the first to find non-collision singularities of the Newtonian n-body problem?

a) Joseph Gerver
b) Jeff Xia
c) Jürgen Moser
d) Henri Poincare

5) If for all t, we have a skew-symmetric matrix $B(t)$ and $S(t)$ satisfies the matrix differential equation $\dot{S} = BS$ with the initial condition $S(0) = I$, then S is

a) orthogonal
b) skew symmetric
c) symmetric
d) the identity matrix

6) Which of the following are ingredients of the proof of chaotic orbits in the Sitnikov problem

a) A horse shoe construction.
b) Stable and instable manifolds
c) The Jacobi integral.
d) The Poincare return map.
e) The Poincare recurrence theorem.
f) Continued fraction expansion.

7) Which of the following statements is called the third Kepler law:

a) The radius vector covers equal area in equal time.
b) Each of the bodies moves on an ellipse.
c) T^2/a^3 is constant.

8) The solar system is a dynamical system which shows very weak type of chaos. If one knows the position of the earth with accuracy $1km$, how long does one have to wait until the uncertainty of the orbit has grown to about 1 astronomical unit (the mean distance of the earth to the sun)?

a) 10'000 years
b) 1 Million years
c) 100 Million years
d) 10 Billion years

9) Which periodic three body solution has been observed in our solar system?

a) Euler motion.
b) Lagrange motion.
c) Hills solutions.
d) Moore choreographies.

10) How many integrals can one find for a general n-body problem:

a) 1
b) 10
c) 3n
d) 6n

11) How many integrals (conserved quantities) did you find for the n-body Toda system?

b) 2
b) n
c) $2n$

12) Which of the following forces occur in a rotating coordinate system and depend on the angular speed of the rotation?

a) Centrifugal force
b) Coriolis force

5/2 QUIZ NR 12	**Math118, O.Knill**

Name:

1) True or False?
A geodesic on a surface connecting two points P and Q is always the shortest path between P and Q.

2) True or False?
For any surface the following is true: between two points, there is exactly one geodesic which connects them.

3) On a surface of revolution, the geodesic flow is integrable. Which of the following integrals are preserved along the orbit? (Several can apply).

a) the Clairot integral
b) the energy integral
c) the momentum integral
d) the angular momentum integral

4) On the hyperbolic plane, a geodesic is

a) either a circle or a vertical line.
b) a line
c) a circle

5) True or False? On any surface of revolution, the longitudinal lines (the intersection with a plane through the axes of rotation) is a geodesics.

6) True or False? On a surface of revolution symmetric with respect to the z-axes, each intersection of the surface with $z = const$ is a geodesic.

7) **Snells law** tells us something about the slope of the shortest connections between two points. Assume to the left of the plane, moving is twice as hard as moving to the right and the angle to the left is α and the angle to the right is β, then

a) $\sin(\alpha) = 2\sin(\beta)$
b) $2\sin(\alpha) = \sin(\beta)$
c) $\sin(\alpha) = \sin(\beta)$
d) $\cos(\alpha) = 2\cos(\beta)$
e) $2\cos(\alpha) = \cos(\beta)$
f) $\cos(\alpha) = \cos(\beta)$

8) True or False? If we have a surface of revolution and we look at the Poincaré map at an "equator" which is the intersection of the surface with a plane perpendicular to the axes of rotation, then a geodesic which starts with an angle $0 < \phi < \pi$ to the equator, then the geodesic curve returns to the equator at some later time.

9) Which of the following equations are called the **Euler-Jacobi equations** for the action functional $I(\gamma) = \int_a^b F(t, x, \dot{x}) \, dt$?

a) $F_p = \frac{d}{dt} F_x$.
b) $F_x = \frac{d}{dt} F_p$.
c) $F_x = -\frac{d}{dt} F_p$.
d) $F_p = -\frac{d}{dt} F_x$.

Hint: If $F(t, x, \dot{x}) = \dot{x}^2/2 - V(x) = T - V$ is the kinetic minus potential energy, then the Euler-Jacobi equations are the Newton equations $\ddot{x} = -V_x$.

10) On which of the following surfaces is every geodesic periodic?

a) The sphere
b) The flat torus
c) The one sheeted hyperboloid
d) The torus embedded in space (the doughnut)