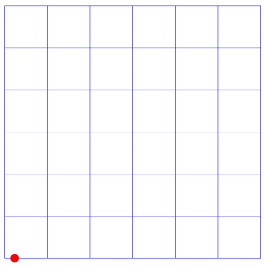
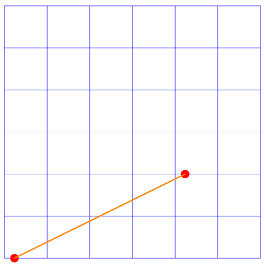
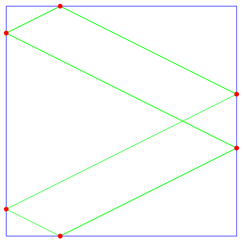
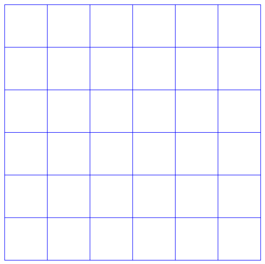
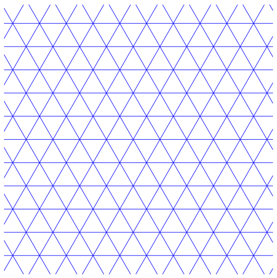
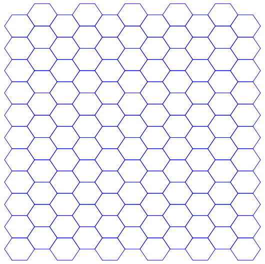
POLYGONAL BILLIARDS **Math118, O. Knill**

ABSTRACT. Billiards in polygons are integrable in the case of rectangles, regular triangles or hexagons.

INTEGRABLE SQUARE. The square and the rectangle are example of an integrable billiard. If θ is the impact angle, then $F(s,\theta) = \sin(2\theta)$ is an integral.

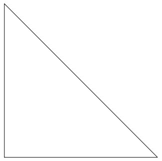
INTEGRABLE POLYGONAL BILLIARDS. If unfolding the polygon produces a tessllation of the plane, the corresponding billiard is integrable.

TRIANGULAR BILLIARDS. Even for triangles, the billiard dynamics is complicated. There are many open questions, one of the most astonishing ones is the open problem:

> Does every triangular billiard have a periodic orbit?

One can solve the problem for the acute triangle? The answer is easy - if you see it.



LETS PLAY SOME GAMES: Lets mention without proof that the Lyapunov exponent of a polygonal billiard is always zero. The chaos, you obtain with these systems is "weak".