6. Homework set	**Math118, O.Knill**

6.1 Let $X = \{0,1\}^Z$ with the distance $d(x,y) = 1/(n+1)$ n is the smallest index such that $x_n \neq y_n$. that is $x_k = y_k$ for $|k| \leq n$ and $x_n \neq y_n$ or $x_{-n} = y_{-n}$.

a) To verify that X is a metric space, have to verify $d(x,y) = d(y,x)$, $d(x,x) = 0$ and the triangle inequality: $d(x,z) \leq d(x,y) + d(y,z)$.

Hint: Actually a stronger inequality holds: $d(x,z) \leq \max(d(x,y), d(y,z))$. If x, y agree on some interval and y, z agree on an other say bigger interval, then x, z agree at least on the smaller interval.

b) Verify that X is compact: every sequence $x(n)$ in X has an accumulation point. You have to show that there is a subsequence $x(k_l)$ such that $x(k_l)$ converges in X for $l \to \infty$.

Hint. You have to construct an accumulation point using a diagonal type argument (unlike in Cantors case, you do not need to change the diagonal).

6.2 Model an epidemic as a CA. You can use your own model. Otherwise, a suggestion would be that people can either be healthy, sick and non-contagious or sick and contagious, The disease is quite severe so that sick people close to a contagious sick person will become sick to. After incubation, they become themselves contagious, then sick and non-contageous, then sick and contagious and then recover again. Describe the CA rule $\phi(a,b,c)$ which gives the state of the person depending on its own state b, and the states a, c of two neighbors. Can you invent a numbering which includes all CA of the type you consider, analogue to the Wolfram numbering over the alphabet $\{0,1\}$?

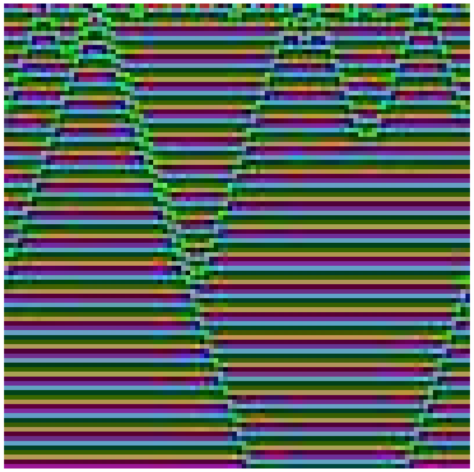
Optional: simulate your CA on the computer.

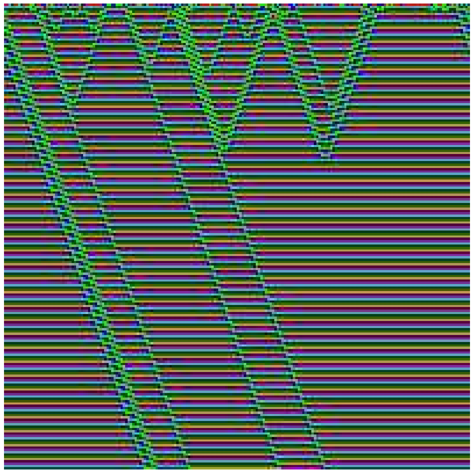
6.3 a) Construct a CA with radius 1 over the alphabet $\{0,1,2,3,4,5\}$ for which the speed c satisfies is $c/R = c = 2/5$.

Remark. If you would generalize your construction to any alphabet, you could proceed to prove that the set c/R of speed/radius ratios of one-dimensional cellular automata is dense in the interval $[0,1]$. Since one can simulate any automaton with alphabet A with an automaton with alphabet $\{0,1\}$ but with an adjustment of the radius, one can see that also in general, the possible speed ratios c/R are dense in the interval $[0,1]$.

6.4 a) Verify that the CA T over the alphabet $A = \{0,1,2\}$ defined by $\phi(x,y,z) = xyz \bmod 3$ has an attractor $K = \bigcap_n T^n(X)$ on which the the map is isomorphic to an elementary CA. (An elementary CA is a CA in one dimensions over the alphabet $\{0,1\}$).

b) (optional) The CA T over the alphabet $A = \{0,1,2\}$ defined by $\phi(x,y,z) = xyz + 1 \bmod 3$ shows "particles". If two particles interact, they annihilate. Can you verify this picture?





6.5 Invent a CA in two dimensions of your choice. Either look at a simple one which you can analyze completely. Or invent one which models some situation (life, forest fire, spread of passion for dynamical systems etc.) It is possible to do experiments on a computer if you wish but this is not necessary. Coming up with an interesting suggestion is also good.