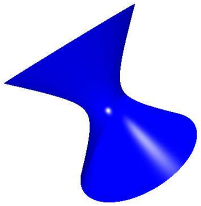
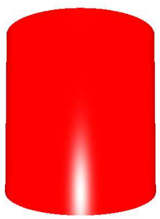
11. homework set	Math118, O.Knill

11.1 Describe the geodesic flow on the one-sheeted hyperboloid. How does a typical geodesic look like. Find one periodic geodesic.

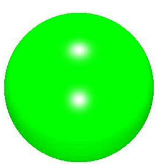


11.2 When you empty a paper towel roll you obtain a cylinder. On this cylinder you find a spiral curve. Prove first that this curve a geodesic. Assume the cylinder is $x^2 + y^2 = 1, 0 \leq z \leq 1$. We play surface billiard: the geodesic curve is reflected at the boundaries $z = 0, z = 1$. Find the return map $T(\theta, \phi) = (\theta_1, \phi_1)$, where $(x, y, z) = (\cos(\theta), \sin(\theta), 0)$ and ϕ is the impact angle.

Optional: From what you know about billiards, can you cut away part of the cylinder to get a chaotic surface billiard?

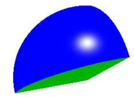


11.3 For a surface of revolution which is symmetric to the xy plane and for which $r(z) \to 0$ for $|z| \to \infty$ we define a Poincare map for the geodesic flow. Sart with a point x on the surface in the xy plane and a unit vector v, follow the geodesic flow along the surface until it comes back to the surface.
This defines return map $T(\theta, \phi)$ on the annulus $(\theta, \phi) \in T \times [0, \pi]$. Describe this map in the case of the sphere.



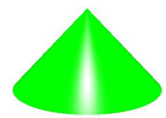
11.4 We play **surface billiard** in the triangular surface obtained by intersecting the unit sphere with the first octant in space. Prove that this billiard is integrable.

Hint. Remember how you analyzed billiards in the rectangle? A similar idea applies here.



11.5 We play **surface billiard** in the half cone $x^2 + y^2 = 1 - z^2$. Prove that there are orbits which never close.

Hint. Similar than for the flat torus or the cylinder one can find geodesics by cutting up the surface and flattening it.



11.6 (optional) Can you prove the statement made in class that on a flat torus, the wave front $K_t(x)$ of a point becomes dense on the torus in the sense, given $\epsilon > 0$, there is a s such that K_t intersects every disc of radius ϵ for $t > s$.

Remark: We do not know whether this statment stays true, if you allow the torus to be bumpy. Nor do we know whether the caustic $C_t(x)$ becomes dense in general. ($C_t(x)$ is the empty set for the flat torus).