Homework 1. Week	**Math118, O.Knill**

1.1 We consider the interval map $f(x) = f_4(x) = 4x(1-x)$. For this particular value, the logistic map is also called the **Ulam map**. We have met it in the first lecture, when we saw that a computer does not "know" the distributative law.

a) Find all the fixed points of the map $T(x) = f(x)$.

b) Analyze the stability of these fixed points. For each point, just tell, whether it is stable or unstable.

c) Draw a graph of this map and start iterating the map using the cobweb construction with the initial value 0.3. Do at least 5 iterates.

1.2 Consider the map $Q_c(x) = x^2 + c$. It is called the **quadratic map**. Again, c is a constant parameter.

a) Verify that this map undergoes a **saddle node bifurcation** (which is also called **blue-sky bifurcation** because of obvious reasons, periodic appear or disappear out of the blue sky. It is also called **tangent bifurcation**). For which value of c does this happen?

b) Analyze the stability of the periodic orbits near the bifurcation value.

c) What happens with the orbits for parameter values c for which we have no fixed point?

1.3 a) Look at the fixed points of the map $Q_c(x) = x^2 + c$ for $c = -0.5$ and determine their stability.

b) Look at the fixed point of the map for $c = -1$ and determine its stability.

c) Verify that the map undergoes a **flip bifurcation** at the parameter $c = -3/4$.

1.4 We define the map $f(x) = 4x + \sin(\pi x)$ mod 1 on the interval $[0, 1]$.

Sideremark: Because after identifying 0 and 1, the interval closes to a circle, the map can be considered a smooth map on the circle. f is an example of a **circle map**.

a) What is the Lyapunov exponent of the orbit of the map f wich an initial condition $x_0 = 1/2$?

b) Verify that the Lyapunov exponent of every orbit of f is positive.

1.5 We have shown that the **Ulam map** $f_4(x) = 4x(1-x)$ is conjugated to the tent map $T(x) = 1 - 2|x - 1/2|$

a) Draw the graph of the the iterates $T^2(x), T^3(x)$ of the tent map. Use the fact that the tent map is piecewise linear.

b) Use the conjugation result to sketch the graphs of the second iterate $f_4^2(x)$ and third iterate $f_4^3(x)$ of the Ulam map.

c) Conclude that f_4^n has 2^n fixed points and therefore, that the Ulam map f has 2^n periodic points of period n.

d) What is the Lyapunov exponent of a periodic point of period n?