

Sample Questions from Past Qualifying Exams

This list may give the impression that the exams consist of a series of questions fired at the student one after another. In fact most exams have more the character of a conversation with considerable give and take. Hence this list cannot be expected to indicate accurately the difficulties involved.

The list indicates the professor associated to each question where available. Some have been in the MGSA files for a while, and this information has been lost (if it was ever there).

The listing by section is approximate, since some questions may fit under more than one heading.

Banach Spaces and Spectral Theory

- What is a nuclear operator? [Coleman]
- Give an example of an integral operator which is nuclear. [Coleman]
- What can you say about the spectrum of a nuclear operator? [Coleman]
 - Could it be the empty set? [Arveson]
- Give an example of an operator on a real Banach space with no spectrum. [Arveson]
- Does the sum of the elements of the spectrum of a nuclear operator converge? [Coleman]
- What is a trace class operator? [Coleman]
- What is a Hilbert-Schmidt operator? Can you give an example over \mathcal{L}^2 of the unit interval? [Coleman]
- Can $[0, 1]$ be the spectrum of a compact operator? [Arveson]
- What is the spectrum of $M_{e^{2\pi it}}$? How could you know that it is invertible? What is the inverse? [Arveson]
- If T is an operator on a Banach space, what is $\cos^2 T + \sin^2 T$? [Arveson]
- What is $\cos T$ [Arveson]
- If f is an entire function, what is fT ? [Arveson]
- List the properties of the functional calculus. [Arveson]
- Consider $\mathcal{C}([0, 1], \mathbb{R})$. Is there a natural topology on this space? [Arveson]
- Let $S = \{f \in \mathcal{C}[0, 1] \mid |f(x) - f(y)| \leq |x - y|\}$. What properties does it have (e.g. closed, complete, bounded compact)? [Arveson]
- Let $S_0 = \{f \in S \mid f(0) = 0\}$. What properties does it have (e.g. closed, complete, bounded, compact) ? [Arveson]
- What is the Riesz theory of compact operators?
- What is a Fredholm operator? Can any Fredholm operator be written as the sum of an invertible operator with a compact operator? What is the Fredholm index? What are its properties? How can you obtain an isomorphism between the abstract index group and the integers?

- Suppose you have an operator x on a Hilbert space such that $x - x^2$ is compact. What can you tell me about it?
- In the previous question, you had a projection in the Calkin algebra, and you showed that it can be lifted to $B(H)$. Can you do the same for a unitary?
- What is the polar decomposition? What can you say about it?