Main observations

- $M$ preserves $F: \mathbb{F}_q^{15} \subset \mathbb{F}_q^{15}$

- $(i, j) = 1$ for any field

- $\Theta$, $\Theta_1$, $\Theta_2$, $\Theta_3$

Geometric Hilbert

$X = \mathbb{P}^1_{/\mathbb{P}} \rightarrow \Theta_1$ finite

Theorem $\Theta_1 = (x, t) \in \mathbb{Q}_q$,

- $s_0 = \mathbb{Q}^2_{/\mathbb{Q}_q}$

Induced action of $\sigma_2(\Theta)$ on $\Theta_1$

$\psi = \text{Sym}^2_{\mathbb{Q}_q}$

$\phi \circ \sigma_2(\Theta)$

$\mathcal{O}(\sigma_1) = \{a \in \mathcal{O} \mid \text{extendible} \}

K = \mathbb{Q}_q(\lambda)$

$U = \{\lambda \mid \lambda \in \mathcal{O}_\lambda \}$

$U^+ = \{\lambda \mid \lambda \in \mathcal{O}_\lambda \}$

$M = \langle I, U^+ \rangle$

$\Theta_0 \Theta_1 = \Theta_2$

$\Theta_0 \Theta_1 = \Theta_2$

$h(x, y) = (a, t) \in \mathbb{Q}_q$
i) We get a continuous rep of \( T \) on \( \mathcal{O}(F) \cong \mathbb{Q}_p < t > \) by the same formula, \( t \) extends to \( M \), and \( \mu = (1, p) \) satisfies \( \mu(t) = pt \) \( \mu(\mathbb{T}^m) = \mathbb{T}^m \) have is compact.

\[ \forall k \in \mathbb{Z}_+, \quad \mathcal{V}_k^\mu = H^0(\mathcal{F}, \mathcal{O}(k)) = \chi^k \mathcal{O}(F) \cong \mathcal{O}(F) \otimes \mathcal{O}(F) \text{ twisted by } \mathcal{O}(k) \]

\[ \forall k \geq 0. \]

And we can recover \( \mathcal{V}_k \): if \( \psi \in \mathcal{O}_p [\mathcal{T}_m \mathcal{I}] \), or \( \psi \in \mathcal{V}_k \) \( (k \geq 0) \)

eigenvalue \( \psi(m) = \lambda m \) with \( \lambda(m) < k+1 \) then \( \psi \) analytically continue to \( X \). (The norm of \( \mu \) on \( \mathcal{V}_k / \mathcal{V}_k \) is \( \leq 1^{k+1} \), clear

ii) If \( X : \mathbb{Z}_p \to L^* \) a continuous character, \( \mathcal{V}_k \) makes sense:

\[ L \otimes \mathcal{O}(F) \otimes \mathcal{O}(F) \text{-twisted action} \]

\[ \to \text{ a pretty new family of rep of } T \text{ parameterized by } \mathcal{V}_k \]

B the big Bruhat-Jwahor cell

\[ X = L^G \text{ flag variety, look at } \mathcal{O}_p [\mathcal{T}_m \mathcal{I}] \text{ on the right.} \]

Lemma

1) \( G = \bigcup_{w \in G_0} \mathcal{L}_w \mathcal{I} \)

\( \mathbb{Z}_p \) big Bruhat cell (open subds.)

\[ \mathcal{L}_w \mathcal{I} \subset \mathcal{L}_B, \quad \text{so} \quad \mathcal{L}_w \mathcal{I} \simeq \mathbb{Z}_p \]

\[ \text{is the } \mathbb{Z}_p \text{-point of an affinoid subdomain of } X_{an} \]

call it \( F \) ("big BI cell")

\[ \frac{\mathbb{A}^{n(n-1)/2}}{\mathbb{A}^{n(n-1)/2}} \]

(especially: \( F \cap \mathbb{A}^{n(n-1)/2} = (\mathbb{Z}_p)_{n(n-1)/2} \))

\[ \frac{n(n-1)}{2} \text{ dim closed polydisc} \]

\[ \frac{n(n-1)}{2} \text{ dim closed polydisc} \]
(i) Use $G = K$ and Bruhat dec. of $GL_n(\mathbb{F}_p)$

(ii) Lower - Upper dec. shows $L^{\mu_{LB}} \cong N$ open subscheme of $X$ (defined by $\Delta_i \neq 0 \quad \forall \ i \in 1, \ldots, n$) & the principal max

- Use $\mathcal{I} \triangleq (L\mathcal{I}) \times (N\mathcal{I})$ (Iwahori - decomp.)

(iii) $\mathcal{I}$ stable by definition

$$(\begin{pmatrix} 1 & a_i^0 \\ 0 & 1 \end{pmatrix}) \sim (\begin{pmatrix} 1 & p_i^0 \cdot a_i^0 \\ 0 & 1 \end{pmatrix}) \in N(\mathbb{Z}_p) \Leftrightarrow a_i^0 \geq a_i \quad \forall \ i \geq 1$$

$$(\begin{pmatrix} 1 & a_i^0 \\ 0 & 1 \end{pmatrix}) \in N(\mathbb{Z}_p) \Leftrightarrow a_i^0 > a_i, \quad \forall \ i > 1$$

**Example**

- $n = 2, \quad X = \mathbb{P}^1, \quad N(\mathbb{Z}_p) \to \mathbb{Z}_p$ \( \triangleleft \) the preceding case
- $n = 3$, action of $\mathcal{I}$ on $\mathbb{P}^2$ a little more complicated already

A simpler way is to view $\mathcal{F} \subset \mathbb{P}(V^*) \times \mathbb{P}(\Lambda^2 V^*)$ (\( V = \mathbb{Q}_p^3 \))

we get $O(\mathcal{F}) \times \mathbb{Q}_p <u, v, x, y>$ incidence relation,

$$(y - ux + v) \not\in \text{Siegel 2nd V} \otimes \text{Siegel 2nd} \Lambda^2 V$$

**Consequences**

Set $O(\mathcal{F}, n) = \mathbb{Q}_p$ algebra functions $F(\mathbb{Q}_p) \to \mathbb{Q}_p$.

$n \geq 0$ integer

analytic on each “$a + \mathbb{Q}_p^0 N(\mathbb{Z}_p)$”

+ sup norm on \( \bigcup_a \text{disc} (a, \mathbb{Q}_p^0) \)

Then $O(\mathcal{F}, n)$ is a projective representation of $M, \mathcal{I}$ acts by isometries,

and $\mathcal{I} \in U^+$ by compact op. of norm $\leq 1$

$\mathcal{I} \in U^+, \ n > 1$, then $O(\mathcal{F}, n) \to O(\mathcal{F}, n)$ is compact.
Weights

3. Integral weights

\[ \mathbb{Z}^n = \{ \mathbf{k} = (k_1, \ldots, k_n) \in \mathbb{Z}^n \mid k_1 \geq \cdots \geq k_n \} \]

For \( k \in \mathbb{Z}^n \), \( \exists ! \) irreducible rep of \( G \), \( \mathbf{V}_k \), such that \( V_k \mathbb{Z}^n \mathbb{Z}^n \) acts through \( \mathbf{V}_k \).

Fundamental rep. \( \Lambda^\text{\_i} \mathbf{V} = V_k \).

Tautologically, \( X \rightarrow \mathbb{P}(\Lambda^\text{\_i} \mathbf{V}) \) with \( \Theta(1) =: L_\Lambda \).

Lemma

i) (Bad. Wul.) \( k \in \mathbb{Z}^n \), \( V_k = \mathcal{H}^0(X, L_\Lambda \otimes \mathcal{O}^k) \mathcal{O}(k) \).

\[ V_k \rightarrow \mathcal{V}_k := \mathcal{H}^0(F, \mathcal{O}(k)) \mathcal{O}(k) \]

ii) each \( L_\Lambda \) is trivial on \( F \) and trivializes (over \( \mathbb{Z}_p \)) by \( \Delta_i \).

iii) each \( \mathcal{V}_k \) is trivial on \( F \) and trivializes (over \( \mathbb{Z}_p \)) by \( \Delta_i \).

iv) \( \forall \mathbf{x} \in \mathbb{P}^{-1}(\mathbb{Z}_p) \), \( \mathbf{x} = (1, r, \ldots, \mathbf{x}^n) \), \( \mathbf{v} \in \mathcal{V}_k \), \( (k \in \mathbb{Z}_p^n) \)

is an eigenvector \( \mathbf{v}(\mathbf{w}) = \lambda \mathbf{w} \) with \( \lambda(\mathbf{w}) = k_{i+1} - k_i - 1 \) for all \( i \),

then \( \mathbf{w} \in \mathcal{V}_k \).

Proof:

i) Well known.

ii) \( \mathcal{F} \) is open and \( X \) is irreducible.

iii) \( L_\Lambda \) is trivial on \( \mathcal{L}(\mathcal{F}) \).

iv) \( \mathcal{V}_k \) is trivial on \( \mathcal{L}(\mathcal{F}) \).

We consider the Plücker embedding

\( X \rightarrow \mathbb{P}(\Lambda^\text{\_i} \mathbf{V}) \)

\[ \mathcal{V}_k \rightarrow \mathcal{V}_k \rightarrow \mathcal{V}_k \rightarrow \mathcal{V}_k \rightarrow \]

\( \mathcal{V}_k \rightarrow \) compute norm of \( (1, \mathbf{x}^1, \ldots, \mathbf{x}^n) \).

\( \mathcal{V}_k \rightarrow \) quotient by \( \mathcal{V}_k \).

and we compute the norm here with \( \mathcal{V}_k \).
Coevdes. Set \( \mathfrak{m}^n = \det \mathfrak{m}^i \). By (\text{iii})\text{), we have \( \varpi \)-convergent \( \mathcal{I} \rightarrow \mathcal{O}(L)^x \) (even on \( \mathbb{Z}_p \))

defined by

\[
\tilde{f}^i (x) = \frac{\gamma (\Delta_i)}{\Delta_i}
\]

("first row of \( \gamma \) acting on \( L^w \)"") looks like at \( \mathfrak{m}^n \)

Fact: Each \( \tilde{f}^i \) extends to a \( \varpi \)-convergent \( \mathcal{I} \rightarrow \mathcal{O}(L)^x \) as that.

\[
\tilde{f}^i (U^+) = 1.
\]

Use that \( M = \bigcup_{w \in V} M_w \). \( M_{w_1} M_{w_2} \subset M_{w_1 w_2} \). Do twist the natural \( \tilde{f}^i \)'s by a character

\[
\psi \in \text{Hom}_{\mathfrak{m} \text{-cont}} (\mathbb{T}(\mathbb{Z}_p), \mathbb{C}^*)
\]

\( \varpi \)-adic character space

fix \( \nabla \in \psi \) open affinoid, universal character \( \psi \) whose restriction to \( (1 + \mathfrak{p}^r \mathbb{Z}_p)^m \) is analytic for some \( r > 0 \)

define \( V_{\nabla, r} : = \mathcal{O}(L)^x \mathbb{C}^{\psi \text{-linear}} \) as \( M \)-module (\( \mathcal{O}(\mathfrak{m}) \)-linear).

Twisted by \( \chi_{\nabla, r} (\tilde{f}^i) \chi_{\nabla, r} (\tilde{f}^i) \cdots \chi_{\nabla, r} (\tilde{f}^i) \chi_{\nabla, r} (\tilde{f}^i) \chi_{\nabla, r} (\tilde{f}^i) \chi_{\nabla, r} (\tilde{f}^i) \chi_{\nabla, r} (\tilde{f}^i) \)

Note that if \( \chi : \mathbb{Z}_p^r \rightarrow \mathbb{A}^r \) is analytic on \( 1 + \mathfrak{p}^r \mathbb{Z}_p \), then

\[
\forall \gamma \in \mathfrak{m} \quad (\tilde{f}^i (\gamma)) : \mathbb{F}(\mathbb{Q}_p) \rightarrow \mathbb{Z}_p^r \rightarrow \mathbb{A}^r
\]

is analytic on the \( \varpi \)-thickens, \( \epsilon \in \left( \mathcal{O}(F, \varpi) \otimes \mathbb{A} \right)^{\times} \).

Prop: \( V_{\nabla, r} \) is an \( \mathcal{O}(L)^x \)-module

- \( \mathcal{I} \) acts continuously, by isometries.
- \( U^{++} \) acts through (constant) compact \( \mathcal{O}(L)^x \)-endomorphisms of norm \( < 1 \).

\( \Box \): clear.